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Plasma dispersion of multisubband electron systems over liquid helium

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Abstract. Density–density response functions are evaluated for nondegenerate multisubband electron systems in the random-phase approximation for arbitrary wavenumber and subband index. We consider both quasi-two-dimensional and quasi-one-dimensional systems for electrons confined to the surface of liquid helium. The dispersion relations of longitudinal intrasubband and transverse intersubband modes are calculated at low temperatures and for long wavelengths. We discuss the effects of screening and two-subband occupancy on the plasmon spectrum. The characteristic absorption edge of the intersubband modes is shifted relatively to the single-particle intersubband separation and the depolarization shift correction can be significant at high electron densities.

1. Introduction

Two-dimensional electron systems (2DES) over the liquid helium surface have been intensively studied for a long time [1]. More recently, it has become possible to confine these surface electrons (SE) in reduced geometries creating also one-dimensional (1D) systems [2]. These systems provide a nearly ideal laboratory for studying collective phenomena in the electron plasma in lower dimensions because the cleanliness of the helium surface restricts SE scattering mechanisms to those with helium atoms in the vapour phase, which predominates at $T > 1$ K, and with surface oscillations (ripplons) at lower temperatures. Furthermore, both scattering mechanisms become ineffective with lowering temperature and can be disregarded at $T \lesssim 0.1$ K. In such a regime, collective effects in low-dimensional electron systems due to the Coulomb interaction can be investigated ignoring the interaction with scatterers. Another important feature of these systems is the accessible range of SE densities which is limited to $n_s \lesssim 10^9$ cm^{-2} (for bulk helium). As a consequence, the 2D Fermi energy $\varepsilon_F \lesssim 10^{-2}$ K and SE behave like nondegenerate low-dimensional systems differing in many respects from the quantum counterparts realized in semiconductor structures [3].

As is well known, the collective excitation spectrum depends crucially on the way in which the particles are confined. For instance, for longitudinal plasma oscillations of the 2DES, the spectrum $\omega_{2D}(q) \sim q^{1/2}$ contrasts with that for the 3D situation in which one has an optical mode starting from the plasma frequency. This is a consequence of the fact that the screening is incomplete in 2D because there are electromagnetic fields in the vacuum surrounding the plane and many-body effects play an important role in governing the properties of the 2DES. On the other hand, the longitudinal plasmon mode in the 1DES case is $\omega_{1D}(q_x) \sim q_x \ln(q_x \ell)$, where ℓ is some characteristic length of the system. In these cases, we have assumed that only the lowest subband, for electron motion along the direction perpendicular to the electron sheet, is occupied. This limiting case is achieved when the Boltzmann factor $\exp(-\Delta_{21}/T) \ll 1$, where $\Delta_{21} = \Delta_2 - \Delta_1$ is the energy gap between the lowest (1) and the first occupied (2) subband, and the occupations of higher subbands are negligible. Otherwise, the multisubband nature of low-dimensional electron systems—hereafter referred to as quasi-2D(1D)ES—cannot be disregarded when the temperature is comparable with Δ_{21} and population effects of higher subbands cannot be ignored.

In this paper, we address the problem of the plasmon spectrum in Q2DES and Q1DES over the surface of liquid helium. We use the many-body dielectric formalism within the random-phase approximation. In this approach, the mode spectrum is obtained from the roots of a determinantal equation for the dielectric function. At first glance, we note that the multisubband character of these systems allows the existence of transverse modes of plasma oscillations in the direction normal to that of unconfined electron motion.

We adopt a two-subband model where the bare electron–electron interaction is evaluated using subband wave functions found by the variational method for the Q2DES and taken as the harmonic-oscillator functions for the parabolic confinement in the Q1DES. We limit ourselves to the case of low enough temperatures, which allows us to disregard the coupling of plasma oscillations with ripplon modes. We also do not consider the possible transition of the electron system to the ordered state where the electron–ripplon interaction can strongly modify the mode spectrum [4–6].

2. Theoretical approach

The main theoretical approach to the study of plasma oscillations in the multisubband low-dimensional charge system is based on the many-body dielectric formalism using the generalized dielectric function

$$\epsilon_{nn',mm'}(\omega, q) = \delta_{nn'}\delta_{mm'} - V_{nn',mm'}(q)\Pi_{mm'}(\omega, q) \tag{1}$$

where $\Pi_{mm'}(\omega, q)$ is the density–density response function, $\delta_{nn'}$ is the Kronecker symbol, and $V_{nn',mm'}(q)$ is the matrix element of the Fourier-transformed Coulomb interaction averaged over wave functions of subbands with indices n, n', m, and m' equal to 1, 2, 3, ... The dielectric function $\epsilon_{nn',mm'}(\omega, q)$ depends both on the frequency and on the wavenumbers q for the Q2DES and q_x for the Q1DES.

In the random-phase approximation (RPA), we assume that the electron system responds to external perturbations as a noninteracting system and we take $\Pi_{mm'}(\omega, q) = \Pi^0_{mm'}(\omega, q)$, where the free polarizability function is written as

$$\Pi^0_{mm'}(\omega, q) = \sum_{k,\sigma} \frac{f_0(E_k + \Delta_m) - f_0(E_{k+q} + \Delta_{m'})}{\hbar\omega + E_k + \Delta_m - E_{k+q} - \Delta_{m'} + \mathrm{i}\delta}. \tag{2}$$

Here $E_k = \hbar^2k^2/2m$, where m is the electron mass, δ is a positive infinitesimal, and σ is the spin index. For classical systems, the distribution function $f_0(E_k + \Delta_n) = \exp[-(E_k + \Delta_n)/T]$ and is normalized by the condition $\sum_{n,k,\sigma} f_0(E_k + \Delta_n) = N$ where N is the number of particles.

Using the dielectric function given by equation (1), Vinter [7] and Das Sarma [8] have studied many-body effects in the degenerate Q2DES. Das Sarma and co-workers [9–11], Hu and O'Connell [12], and Hai *et al* [13] extended these studies to plasma oscillations in degenerate Q1D multisubband systems whereas Sokolov and Studart [14] approach the problem in the classical regime.

The well-known bare electron–electron potential is given by

$$V^{2D}_{nn',mm'}(q) = \int_0^\infty \int_0^\infty \mathrm{d}z\,\mathrm{d}z'\; \psi_n(z)\psi_{n'}(z)v^{2D}(q)\psi_m(z')\psi_{m'}(z') \tag{3}$$

and

$$V^{1D}_{nn',mm'}(q_x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}y\,\mathrm{d}y'\,\varphi_n(y)\varphi_{n'}(y)v^{1D}(q_x)\varphi_m(y')\varphi_{m'}(y') \qquad (4)$$

where $v^{2D}(q) = 2\pi\tilde{e}^2/Sq$ ($v^{1D}(q_x) = 2(\tilde{e}^2/L_x)K_0(|q_x||y - y'|)$) is the Coulomb potential, S (L_x) the area (length) of the system, and $\psi_n(z)$ ($\varphi_n(y)$) denote the nth-subband wave functions for the Q2DES (Q1DES). Here $\tilde{e} = [2e^2/(1+\varepsilon)]$, with ε the helium dielectric constant, is the effective charge taking substrate effects into account.

3. Plasmon spectrum

The dispersion relations for collective modes for a multisubband system are found from the roots of the determinantal equation

$$\det|\epsilon_{nn',mm'}(q,\omega)| = 0. \qquad (5)$$

In principle, all the subbands should be considered in the above equation. However, a useful analytical solution is possible in a two-subband model. In this case, equation (5) splits into two independent equations:

$$1 - V_{11,11}\Pi^0_{11}(\omega, q) = 0 \qquad (6a)$$

$$1 - V_{12,12}\left[\Pi^0_{12}(\omega, q) + \Pi^0_{21}(\omega, q)\right] = 0. \qquad (6b)$$

Mode coupling appears only if one takes into account higher subbands. Equation (6*a*) describes the longitudinal *intrasubband* plasma oscillations whose dispersion law must coincide with that of the 2DES or 1DES with a one-subband occupancy system whereas equation (6*b*) gives the dispersion law for transverse *intersubband* oscillations involving transitions from the lowest to the second subband.

3.1. Q2DES

As is well known, SE on helium are trapped in the direction perpendicular to the surface (the z-direction) by a potential well due to image forces and a holding electric field $E_\perp$. For $E_\perp = 0$, and the image potential $V(z) = -\Lambda_0/z$, where $\Lambda_0 = (e^2/4)(\varepsilon - 1)/(\varepsilon + 1)$, and an infinite potential barrier at the interface, the solution of the Schrödinger equation is given by [15–17]

$$\psi_n(z) = \frac{2\kappa_0^{3/2} z}{n^{5/2}} \exp\left(-\frac{\kappa_0 z}{n}\right) L^{(1)}_{n-1}\left(\frac{2\kappa_0 z}{n}\right) \qquad (7)$$

where $\kappa_0 = m\Lambda_0/\hbar^2$ ($=3/(2\langle z\rangle_1)$), where $\langle z\rangle_1$ is the mean electron distance from the plane) and $L^{(\alpha)}_n(x)$ are the associated Laguerre polynomials. The energy subband is given by the hydrogen-like spectrum $\Delta_n = -\Delta_0/n^2$ where $\Delta_0 = \hbar^2\kappa_0^2/2m$. If the pressing electric field $E_\perp$ is turned on, there is no general analytical solution and one should use approximate methods. If $E_\perp$ is small ($\lesssim$1.5 kV cm^{-1} for $n = 1$), the subband energies can be evaluated perturbatively resulting in the linear Stark effect contribution $eE_\perp\langle z\rangle_n = 3eE_\perp n^2/2\kappa_0$, which was seen experimentally [18, 19]. A simplified analysis of the wave functions and energy gaps gave excellent agreement with experimental data. At large electric fields, one can discard the image potential to obtain $\Delta_n = |\zeta_n|eE_\perp\kappa_\perp^{-1}$, with $\kappa_\perp = (2meE_\perp/\hbar^2)^{1/3}$ and the ζ_n being the zeros of the Airy function. In order to evaluate an approximate solution over a wide range of $E_\perp$, we assume trial wave functions corresponding to the two lowest subbands ($n = 1$ and 2) of equation (7) with variational parameters κ_1 and κ_2 [20, 21]:

$$\psi_1(z) = 2\kappa_1^{3/2} z \exp(-\kappa_1 z) \qquad (8a)$$

$$\psi_2(z) = \frac{2\sqrt{3}\kappa_2^{5/2}}{\kappa_{12}}\left[1-\left(\frac{\kappa_1+\kappa_2}{3}\right)z\right]z\exp(-\kappa_2 z) \tag{8b}$$

and subband energies

$$\Delta_1 = \frac{\hbar^2\kappa_1^2}{2m} - \Lambda_0\kappa_1 + \frac{3eE_\perp}{2\kappa_1} \tag{8c}$$

$$\Delta_2 = \frac{\hbar^2\kappa_2^2}{6m}\left[1+\frac{6\kappa_2^2}{\kappa_{12}^2}\right] - \frac{\Lambda_0\kappa_2}{2}\left[1+\frac{2\kappa_2^2-\kappa_1\kappa_2}{\kappa_{12}^2}\right] + \frac{eE_\perp}{2\kappa_2}\left[1+\frac{4\kappa_1^2-\kappa_1\kappa_2+\kappa_2^2}{\kappa_{12}^2}\right]. \tag{8d}$$

Here $\kappa_{12}^2 = \kappa_1^2 - \kappa_1\kappa_2 + \kappa_2^2$. If we define $\kappa_1(E_\perp) = \eta_1\kappa_0$ and $\kappa_2(E_\perp) = \eta_2\kappa_0$, we can find η_1 and η_2 as the roots of the system of equations given by

$$\eta_1^3 - \eta_1^2 - \frac{3}{4}\left(\frac{\kappa_\perp}{\kappa_0}\right)^3 = 0 \tag{9a}$$

$$\eta_2^3(\eta_1^4 - 2\eta_1^3\eta_2 + 15\eta_1^2\eta_2^2 - 11\eta_1\eta_2^3 + 7\eta_2^4) - \frac{3}{2}\eta_2^2(\eta_1^4 - 4\eta_1^3\eta_2 + 10\eta_1^2\eta_2^2 - 6\eta_1\eta_2^3 + 3\eta_2^4)$$
$$- \frac{3}{4}\left(\frac{\kappa_\perp}{\kappa_0}\right)^3(5\eta_1^4 - 10\eta_1^3\eta_2 + 15\eta_1^2\eta_2^2 - 4\eta_1\eta_2^3 + 2\eta_2^4) = 0. \tag{9b}$$

For $E_\perp = 0$, equations (9*a*) and (9*b*) reproduce the results given by equation (7) and the respective eigenenergies with $\eta_1 = 1$ and $\eta_2 = 0.5$. In the variational approach, the gap energy Δ_{21} exhibits a rapid increase at low field and an asymptotic linear behaviour at very large fields. For comparison, Δ_{21} is 40% larger than the corresponding value in the strong-field limit $(|\zeta_2| - |\zeta_1|)eE_\perp\kappa_\perp^{-1}$, when the image potential is disregarded.

Using equations (3), (8*a*), and (8*b*) one can calculate the values of $V_{11,11}$ and $V_{12,12}$ up to second order in the parameters $q/\kappa_1 \ll 1$ and $q/(\kappa_1+\kappa_2) \ll 1$ as

$$V_{11,11} = v^{2D}(q)\left[1 - \frac{3q}{4\kappa_1} + \frac{3q^2}{4\kappa_1^2}\right] \tag{10a}$$

$$V_{12,12} = v^{2D}(q)\alpha(E_\perp)\frac{q}{\kappa_0}\left[1 - \frac{16q}{5(\kappa_1+\kappa_2)} + \frac{7q^2}{(\kappa_1+\kappa_2)^2}\right] \tag{10b}$$

with $\alpha(E_\perp) = 60\eta_1^3\eta_2^5/[(\eta_1+\eta_2)^7(\eta_1^2 - \eta_1\eta_2 + \eta_2^2)]$. The well-behaved form of $\alpha(E_\perp)$ does not strongly influence $V_{12,12}$ because $\alpha(E_\perp = 0) = 0.146$ and $\alpha(E_\perp)$ increases on increasing $E_\perp$ until it reaches a maximum $\alpha_{\max} = 0.281$ near $E_\perp = 0.3$ kV cm^{-1} and slightly decreases to 0.227 at $E_\perp = 3$ kV cm^{-1}.

For the Q2DES, the noninteracting density–density response function, equation (2), can be calculated in a straightforward way. The result is

$$\Pi^0_{nn'}(\omega, q) = -\frac{N}{\hbar q u_T Z_n}\left[\exp(-\Delta_n/T)U(\zeta_{nn'}^{(-)}) - \exp(-\Delta_{n'}/T)U(\zeta_{nn'}^{(+)})\right] \tag{11}$$

where $\zeta_{nn'}^{(\pm)} = [\omega + (\Delta_n - \Delta_{n'})/\hbar]/qu_T \pm \hbar q/2mu_T$, with $u_T = \sqrt{2T/m}$ being the thermal velocity and $Z_n = \sum_n \exp(-\Delta_n/T)$. Similar general structure of $\Pi^0_{nn'}(\omega, q)$ is found in the classical regime of the electron gas in the 3D case [22]. The function $U(\zeta)$ is given by the integral

$$U(\zeta) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\exp(-y^2)}{y-\zeta-\mathrm{i}\delta}\,\mathrm{d}y = -2\exp(-\zeta^2)\int_0^\zeta \exp(t^2)\,\mathrm{d}t + \mathrm{i}\sqrt{\pi}\exp(-\zeta^2). \tag{12}$$

For $n = n'$ and for small $q \ll (2m\omega/\hbar)^{1/2}$, equation (11) can be approximately expressed through $W(\zeta)$, the well-known function in the plasma theory, as [23–26]

$$\Pi^{(0)}_{nn}(\omega, q) \simeq -\frac{N}{TZ_n} W\left(\frac{\omega}{qu_T}\right) \exp\left(-\frac{\Delta_n}{T}\right). \tag{13}$$

The function $W(\zeta)$ is connected with $U(\zeta)$ by the relation

$$W(\zeta) = -(\partial U/\partial\zeta)/2 = 1 + \zeta U(\zeta).$$

Putting $\omega = \omega(q) - \mathrm{i}\gamma_q$, and assuming $\omega(q)/qu_T \gg 1$ and $|\omega(q) - \Delta_{21}/\hbar|/qu_T \gg 1$, we obtain, in the two-subband model, the dispersion relation for the longitudinal intrasubband mode frequencies (ω_l) and damping (γ_l):

$$\omega_l^2(q) = \omega_{2D}^2(q)\left[1 + \frac{3q}{k_D} - \frac{3q}{4\kappa_1}\right] \tag{14a}$$

$$\gamma_l(q) = \sqrt{\pi}\frac{\omega_l^4(q)}{(qu_T)^3} \exp\left[-\frac{\omega_l^2(q)}{(qu_T)^2}\right] \tag{14b}$$

where $\omega_{2D}^2(q) = (2\widetilde{e}^2/ma^2)q$ and $k_D = 2\widetilde{e}^2/Ta^2$ is the 2D Debye wavenumber, and $a = (\pi n_s)^{-1/2}$ is the mean interelectron spacing. One can see, from equation (14*a*), that the first two terms of the real part of the longitudinal branch are the same as in the classical 2DES [23–25]. The extra term $(3q/4\kappa_1)$ comes from the effect of the layer thickness, because κ_1 is related to the distance of the electron in the lowest subband from the plane, and lowers the dispersion relation at small wavelengths. From equation (14*a*), we also observe the competition between screening and layer thickness effects on the Q2DES. The Landau damping $\gamma_l(q)$ of the plasma oscillation is practically the same as in the 2DES.

For transverse intersubband modes, we obtain the frequencies (ω_t) and damping (γ_t):

$$\omega_t^2(q) \simeq \omega_{21}^2 + 2\omega_{21}\omega_{2D}^{(sh)}\left[1 + \frac{(1 + 3\omega_{2D}^{(sh)}/2\omega_{21})Tq^2}{m(\omega_{2D}^{(sh)})^2} - \frac{16q}{\kappa_1 + \kappa_2}\right] \tag{15a}$$

$$\gamma_t(q) \simeq \sqrt{\pi}\frac{[\omega_t(q) - \omega_{21}]^2}{qu_T} \exp\left[-\frac{[\omega_t(q) - \omega_{21}]^2}{(qu_T)^2}\right] \tag{15b}$$

where $\omega_{21} = \Delta_{21}/\hbar$ and $\omega_{2D}^{(sh)} = 2\alpha(E_\perp)\widetilde{e}^2/\hbar\kappa_0 a^2$. As one can see, the transverse plasmon mode spectrum, given by equation (15*a*), is quite different from the longitudinal branch and has a gap at $q = 0$. The frequency of the characteristic absorption edge is shifted, relatively to the frequency ω_{12} of the intersubband transition, by

$$\Delta\omega = \sqrt{\omega_{21}^2 + 2\omega_{21}\omega_{2D}^{(sh)}} - \omega_{21}$$

which is the manifestation of the depolarization shift effect in transverse oscillations of the many-body system [12]. The experimental observation of $\Delta\omega$ should be very interesting in evidencing the role of Coulomb effects in the collective electron motion along the z-direction. Note that for the 2D plasma parameter $\Gamma = \widetilde{e}^2/aT \lesssim 1$, $\omega_{2D}^{(sh)} \ll \omega_{21}$ and $\Delta\omega \simeq \omega_{2D}^{(sh)}$, i.e. the absorption edge is very close to ω_{21} being only slightly shifted to higher frequencies. On increasing q, $\omega_t(q)$ decreases according to the last term in brackets in equation (15*a*). However, our estimates show that, for $T \sim 0.1$–1.0 K and $q \sim 10$–10^2 cm^{-1}, the coefficient of the quadratic term is larger than that of linear one. However, in the long-wavelength limit, these coefficients are so small that

$$\omega_t(q) \simeq \omega_{21}\sqrt{1 + 2\omega_{2D}^{(sh)}/\omega_{21}}.$$

As in the longitudinal mode, $\gamma_t(q)$, given by equation (15*b*), is exponentially small, such that $|\gamma_t(q)| \ll \omega_t(q)$.

The absorption edge of the mode $\omega_t(q)$ depends strongly on ω_{21}. For very small $E_\perp$, the experimental values of ω_{21} are close to $3\Delta_0/4\hbar$ and increase linearly with $E_\perp$ [28]. For arbitrary $E_\perp$, ω_{21} is obtained from the gap energy and is in the range of 100 GHz to 1 THz. The polarization shift $\Delta\omega \sim \omega_{sh}^{(2D)} \sim n_s$ for $\Gamma < 1$, even though $|\Delta\omega| \ll \omega_{21}$. For example for $n_s = 10^6$ cm^{-2} and $E_\perp = 0$, we estimate $\omega_{sh}^{(2D)} \sim 100$ MHz $\ll \omega_{21}$. This makes the direct experimental observation of the depolarization shift at this electron density very difficult. However, the effect should be observable at higher densities (for instance $n_s \sim 10^8$ cm^{-2}) and the depolarization shift should be measured for the Q2DES on the helium surface.

3.2. Q1DES

We now consider plasma oscillations in the Q1DES created along a channel filled with liquid helium. As in previous work [14, 29], we consider a parabolic confinement $U(y) = m\omega_0^2 y^2/2$ with the frequency $\omega_0 = (eE_\perp mR)^{1/2}$, where R is the curvature radius of the liquid in the channel. Typical values of R vary from 10^{-4} to 10^{-3} cm [30]. The spectrum of electron subbands along the y-axis is $E_n = \hbar\omega_0(n - 1/2)$, $n = 1, 2, 3, \ldots$, in addition to the subbands along the z-direction. The motion along the x-direction (the channel axis) is free. The frequency ω_0 increases with $E_\perp$ achieving 100 GHz at $E_\perp = 3$ kV cm^{-1} for $R = 5 \times 10^{-4}$ cm. As $\omega_0 \ll \omega_{21}$, the multisubband system in transverse directions can be decoupled and we ignore electron transitions in the z-direction which are the same as discussed above.

The noninteracting density–density response function was calculated in reference [14]. The result was

$$\Pi^0_{nn'}(\omega, q_x) = -\frac{2N\{\exp[-(n-1)\hbar\omega_0/T]U(\zeta^{(-)}_{nn'}) - \exp[-(n'-1)\hbar\omega_0/T]U(\zeta^{(+)}_{nn'})\}}{\hbar q_x u_T\,[1 + \coth(\hbar\omega_0/2T)]} \tag{16}$$

where $\zeta^{(\pm)}_{nn'} = (\omega/q_x u_T)[1 + (\omega_0/\omega)(n - n')] \pm \hbar q_x/2mu_T$. For $\hbar\omega_0 \gg T$, when only the lowest subband ($n = 1$) is occupied, the expression for the response function is greatly simplified yielding $\Pi^0_{nn'}(\omega, q_x) = -(N/T)W(\omega/q_x u_T)$.

Using the wave functions of the two lowest subbands ($n = 1$ and $n = 2$),

$$\varphi_1(y) = \frac{1}{\pi^{1/4} y_0^{1/2}} \exp\left(-\frac{y^2}{2y_0^2}\right) \qquad \varphi_2(y) = \frac{\sqrt{2}}{\pi^{1/4} y_0^{3/2}}\, y \exp\left(-\frac{y^2}{2y_0^2}\right)$$

where $y_0 = (\hbar/m\omega_0)^{1/2}$, we obtain the matrix elements of the Coulomb interaction from equation (4):

$$V^{1D}_{11,11}(q_x) = \frac{\tilde{e}^2}{L_x} \exp\left(\frac{q_x^2 y_0^2}{4}\right) K_0\left(\frac{q_x^2 y_0^2}{4}\right) \simeq \frac{\tilde{e}^2}{L_x} \ln\frac{1}{|q_x y_0|^2} \qquad \text{for } |q_x y_0| \ll 1 \tag{17a}$$

and

$$V^{1D}_{12,12}(q_x) = \frac{\tilde{e}^2}{2L_x} \exp\left(\frac{q_x^2 y_0^2}{4}\right)\left[K_0\left(\frac{q_x^2 y_0^2}{4}\right) - \frac{\sqrt{\pi}}{\sqrt{2}|q_x y_0|} W_{-1,0}\left(\frac{q_x^2 y_0^2}{2}\right)\right]$$
$$\simeq \frac{\tilde{e}^2}{L_x}\left[1 - \frac{q_x^2 y_0^2}{2} \ln\frac{1}{|q_x y_0|}\right] \qquad \text{for } |q_x y_0| \ll 1 \tag{17b}$$

where $W_{\alpha,\beta}(x)$ is the Whittacker function.

Using equations (5), (6*a*), and (17*a*), taking $\omega = \omega(q_x) - i\gamma_q$, and assuming that $\omega(q_x)/q_x u_T \gg 1$ and $|\omega(q_x) - \omega_0|/q_x u_T \gg 1$, we obtain the dispersion relation for the longitudinal intrasubband modes in the long-wavelength limit, $|q_x y_0| \ll 1$, as

$$\omega_l(q_x) = \frac{2\widetilde{e}^2 q_x^2}{m\ell} \ln \frac{1}{|q_x y_0|} \exp\left(\frac{q_x^2 y_0^2}{4}\right) \left[1 + \frac{3T\ell}{2\widetilde{e}^2} \ln^{-1} \frac{1}{|q_x y_0|}\right] \tag{18a}$$

$$\gamma_l(q_x) = \sqrt{\pi} \frac{\omega_l^4(q_x)}{(q_x u_T)^3} \exp\left[-\frac{\omega_l^2(q_x)}{(q u_T)^2}\right] \tag{18b}$$

where $\ell \simeq n_l^{-1} = (N/L_x)^{-1}$ is the mean interelectron distance along the channel.

The longitudinal spectrum mode, given by equation (18*a*), has the same structure as that obtained previously in reference [14] and in references [31, 32] where a quasi-crystalline approximation was employed. However, we found an additional second term in the brackets, which should be quite small for reasonable values of T and ℓ. Note also that the condition $\omega/q_x u_T \gg 1$ assumed here is equivalent to $T \ll e^2/\ell$ in the quasi-crystalline approximation. It is worth emphasizing that the present result was obtained within the RPA which is valid in the opposite limit $T \gg e^2/\ell$. Our conclusion is that the plasmon spectrum in the classical Q1DES has little dependence on the plasma parameter and that RPA results should probably be correct over a wide range of electron densities.

The transverse branch of collective excitations is rather interesting. Following the same steps as before, we arrive at

$$\omega_t^2(q_x) = \omega_0^2 - \frac{\widetilde{e}^2 q_x^2}{m\ell} \ln \frac{1}{|q_x y_0|} + 2\omega_0 \omega_{1D}^{(sh)} \left[1 + \left(\frac{(1 + 3\omega_{1D}^{(sh)}/2\omega_0)T}{m(\omega_{1D}^{(sh)})^2} + \frac{(1 + \omega_{1D}^{(sh)}/\omega_0)\hbar}{2m\omega_{1D}^{(sh)}}\right) q_x^2\right] \tag{19a}$$

$$\gamma_t(q_x) = \sqrt{\pi} \frac{[\omega_t(q_x) - \omega_{21}]^2}{q_x u_t} \exp\left[-\frac{[\omega_t(q_x) - \omega_{21}]^2}{(q_x u_t)^2}\right]. \tag{19b}$$

Here $\omega_{1D}^{(sh)} = \widetilde{e}^2/\hbar\ell$. The first two terms in equation (19*a*) correspond to the result obtained in the quasi-crystalline approximation [31, 32] if y_0 is replaced by ℓ in the logarithmic factor. The next term is the depolarization shift correction increasing the absorption edge frequency by

$$\Delta\omega = \sqrt{\omega_0^2 + 2\omega_0\omega_{1D}^{(sh)}} - \omega_0 \simeq \omega_{1D}^{(sh)}$$

when $\omega_{1D}^{(sh)} \ll \omega_0$. One can see that the instability of the transverse mode ($\omega_t^2(q_x) < 0$) in the limit of zero confinement ($\omega_0 = 0$) is still manifested in our treatment. We call attention to the fact that we found a quite different result in our previous work [14] because we used an approximate expression, $V_{12,12}^{1D}(q_x) \simeq \widetilde{e}^2/L_x$, considered in reference [12]. One estimate is that the polarization shift correction should be quite small for $n_l \sim 10^2$–10^3 cm^{-1} and $T \simeq 10^{-1}$–1 K such that $e^2/\ell < T$. For instance, $\omega_{1D}^{(sh)} \simeq 10$ GHz for $E_\perp = 3$ kV cm^{-1} and $n_l = 10^2$ cm^{-1} whereas $\omega_0 = 100$ GHz at $R = 5 \times 10^{-4}$ cm. However, this density range cannot be achieved under experimental conditions. For higher densities, $\Delta\omega$ should be of the same order as ω_0 and the polarization shift should be observed.

4. Concluding remarks

In the present work, we have used the many-body dielectric formalism to calculate the spectrum of plasma oscillations for the classical Q2DES and Q1DES formed on the liquid helium surface. We have obtained the general expression for the density–density Q2D and Q1D response functions for any frequency and wavenumber within the RPA. The results are valid at low temperatures since we have used a two-subband model in which only the lowest subbands of the motion in the direction normal to the electron layer (Q2D) and of the motion in the direction across the conducting channel (Q1D) are occupied. The plasma dispersion relations were found from the zeros of the determinantal equation for the generalized multisubband dielectric functions. We have obtained corrections to the gapless longitudinal modes, beyond the $q^{1/2}$-behaviour in the Q2DES and sound-like behaviour, within logarithmic accuracy, in the Q1DES. The intersubband transverse collective frequency is higher than the corresponding single-particle excitation frequency both in the Q2DES and in the Q1DES. The absorption edge frequencies are increased by the depolarization shift which can be large at high densities. In this connection, experimental study of intersubband transitions in low-dimensional electron systems over liquid helium looks attractive, because of the accessibility of a wide range of charge concentrations and low temperatures, for observing collective effects on spectroscopic transitions [28].

We conclude by pointing out some limitations of our approach. The results are based on the RPA, which works quite well at small values of the plasma parameter. We know that RPA results become less good as the dimensionality is reduced, but we do not know how to go beyond the RPA in a controlled way particularly in the Q1DES. We are, however, encouraged by the good agreement of our RPA results for the mode spectrum and those obtained in the quasi-crystalline approximation that is valid in the opposite limit of high plasma parameter. Another feature is the excellent agreement between the RPA theory and experiment on collective excitations in semiconductor quantum wells [33] and wires [34]. Our use of a two-subband model should be improved on in more realistic calculations [35]. But we do not expect the correction of including other subbands to be qualitatively significant at low temperatures.

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